Dynamic Internet Pricing and Bandwidth Guarantees with Nash Equilibrium

Rohit Tripathi and Gautam Barua
Department of Computer Science and Engineering
Indian Institute of Technology Guwahati
Guwahati-781039, India
Email: {t.rohit,gb}@iitg.ernet.in

***Abstract*—The decrease in price of internet access has brought clients' focus on a good internet service. Clients now demand guarantees for the bandwidth promised. We present a scheme in which clients are assured connection and bandwidth and if assured service is not provided, service providers pay penalty to them. The use of multi-SIM handheld devices such as mobiles and tablets have enabled clients to make a "choice" between multiple service providers for communication services. With competition, a client will prefer to connect to that provider who is providing the lowest price with guarantees. In this paper, we present a scenario of dynamic pricing and guarantees with penalties and providers have to decide on pricing strategies which will maximize their income. We present a solution among two service providers which achieves a Nash equilibrium with the maximizing of the expected income being the decision criterion.**

I. INTRODUCTION

The decrease in price of internet access has brought clients' focus on a good internet service. Clients want to have some guarantee on quality of service (QoS) of internet access. An alternate to providing hard guarantees and getting no fees if guarantees are not kept, is to provide for penalties if assured quality is not provided on a dynamic basis. In this paper, we present a model with bandwidth guarantees and penalties if assured service is not provided.

The use of multi-SIM handheld devices such as mobiles have enabled clients to get network access from more than one service provider. We assume that such access is for internet access in this paper. When clients want a connection, they have to choose from one among a number of possible Internet service providers (ISPs). If instead of randomly choosing a service provider, they choose "the most appropriate" ISP, the quality of service is likely to be better, and traffic may also get balanced among the providers. We consider a system of multihomed clients and two ISPs in which each client can connect to the two ISPs. A service provider wants to maximize its income by charging appropriate prices and reducing penalty paid to clients. We present Nash equilibrium solutions for two service providers that maximize the income of service providers.

The rest of the paper is as follows. The related work is mentioned in section II. In section III, we introduce research problem and present solution. In section IV, approximate solution is presented. We conclude the paper in section V.

II. RELATED WORK

There are many pricing schemes in Wireless Networks [2]. In most of the existing pricing schemes, service providers decide static or dynamic prices and clients decide to access the service based on the prices. A service provider chooses appropriate strategies to maximize its income. This means that when a service provider has to choose between one of two alternatives, in one of which it earns more money and in the other it provides better service to its clients, the service provider will surely choose the first alternative. There should be some way to force service providers to provide good service. This can be done using service level agreements. In [5], such a service level agreement framework is mentioned. If service providers fail to provide the QoS promised, they pay a penalty to clients. In [5], a client after connection is assured some bandwidth and whenever a client wants the bandwidth, he should get it immediately, otherwise the provider pays a penalty. A client is allowed to release some or all of the assured bandwidth and pay less. The service provider does not provide connection guarantees. The service provider has to maximize his income and do admission control. The solution mentioned is only heuristic based. In [3], authors have mentioned different types of penalty functions. However, there is no analysis of their scheme.

A lot of work has been done on game theory based network selections [6]. In [4], authors have presented a non-cooperative game among service providers and clients who have access to multiple service providers. Service providers use dynamic pricing to maximize their income. Clients choose an appropriate service provider based on different factors like price, service provider's reputation, mobility etc. In our model a client uses only price to choose a service provider. Our model assumes that a client after connection takes service from one service provider for the complete connection duration, and a connected client can sometimes remain idle and sometimes consume bandwidth.

III. RESEARCH PROBLEM AND SOLUTION

We assume that there are two service providers and each client can connect to either of the two ISPs. Each client requires exactly one unit of bandwidth (this is to reduce the size of the solution space). When a client tries to connect, each of his service providers announces a price from a finite set of possible prices. The client connects to the ISP offering the lower price, randomly choosing one if the prices are the

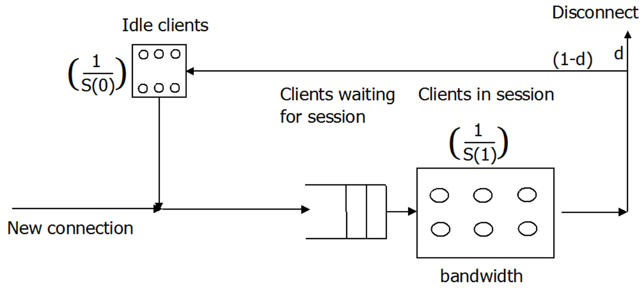

Figure 1. Connected clients

same. If an ISP decides to reject a request, it offers the highest possible price. This is done to avoid paying a penalty without a client deciding to connect to it. So, if ISP 1 wants to reject and ISP 2 is offering the highest price, the client will see both offering the highest price and it is only with a 50 % probability that ISP 1 will have to pay a penalty as the client may connect to ISP 2 and get accepted. So only after an ISP is chosen, will the ISP accept or reject the connection. In the latter case, the ISP will pay a penalty to the client. The client will then connect to the second ISP. Here again there may be an acceptance or a rejection.

As already mentioned, we assume that a service provider can charge prices from a finite set of discrete prices. Competition wwill ensure that prices do not vary much, and yet, the prices will be sufficiently far apart to make a difference.

The arrival of clients is modelled as a Poisson process[1]. λ is the mean arrival rate of clients. After connection, a client sometimes remains idle and sometimes consumes bandwidth and these durations are both exponentially distributed. When a client consumes bandwidth, it is termed as being in session. When a client requests for bandwidth but he has to wait, it is termed as waiting for session. When a connected client is neither in session nor waiting for session, he is in an idle state. A client moves out of idle state when he needs to enter the session state. He may have to wait to get into the session state. If so, he gets a penalty from the ISP for the duration he has to wait. He is then waiting for a session. Figure 1 illustrates the scheme. After a session ends, with probability d the client exits the system. Otherwise, he goes into the idle state.

Consider a situation in which two service providers have the knowledge of the parameters (bandwidth, prices, etc.) of each other. Assume service provider 1 chooses strategy $st1$ and service provider 2 chooses $st2$. If the best strategy of service provider 1 is $st1$ given that the strategy of service provider 2 is $st2$ and the best strategy of service provider 2 is $st2$ given that the strategy of service provider 1 is $st1$, then this situation is said to be in Nash Equilibrium. If the two service providers are in Nash equilibrium, they are likely to be doing their best. Our problem is to find a Nash Equilibrium between two service providers.

A. Symbol Declaration

The state of a service provider is represented by a set of integers (m, n) where m is the number of clients connected and n is the sum of the number of clients in session and the number of clients waiting for a session. It is shortened to a single symbol s. The definitions of all the symbols used are given in Table I.

Table I. SYMBOL DECLARATION

Symbol	Definition
$state$	It represents the state of a service provider which is the set (m, n). As a shorthand, it is represented by s, $s1$, or $s2$ below.
m	Number of connected clients of the service provider being considered.
n	Sum of the number of clients in session and the number of clients waiting for session of the service provider being considered.
m_{max}	Maximum possible value of m.
d	Probability that a client disconnects immediately after closing a session.
$\frac{1}{S(i)}$	If i is zero, it is the mean duration for which a client remains idle and then requests for bandwidth and if i is 1 it is the mean duration for which a client consumes 1 unit bandwidth and then releases bandwidth.
$price()$	An array consisting of possible prices a service provider can charge. The values are in ascending order. $price(0)$ is the least price.
$C(i, s)$	The decision service provider i takes when the state of a service provider is (s). When $C(i, s)$ returns zero or less, the client is accepted. When $C(i, s)$ returns 1, the client should be rejected. When $C(i, s)$ is zero or less a new arriving client is charged the price $price(-C(i, s))$.
$Pr(s1, s2)$	Probability that the state of service provider 1 is $s1$ and the state of service provider 2 is $s2$.
λ	The mean arrival rate of clients.
B_i	Bandwidth of service provider i.
T	Number of prices.
D	Expected total data transferred by an arriving client.
$I(i)$	Income per unit time at steady state of service provider i.
$P(0)$	Penalty per unit time paid to a client waiting for bandwidth.
$P(1)$	Penalty paid to the client being rejected.
$Pri(m, n)$	Finite population steady state probability when the population is m and $(m - n)$ clients are idle. In other words, n is the sum of the number of clients in session and the number of clients waiting for session
$E(i, m)$	The expected number of clients in session for service provider i when m clients are connected and so the population is finite
$Delay(i, s)$	The expected delay per unit time for service provider i when state of service provider is s

B. Nash Equilibrium solution method (Accurate Solution)

Our solution finds a Nash equilibrium if it exists. Our payoff function (or the criterion for deciding if a solution is the best) is the expected income of a service provider at steady state.

Solution steps : The method of finding a Nash equilibrium has the following steps. We use variable x and y to denote the two service providers. To begin with, let x be service provider 1 and y be service provider 2.

1) Start with service provider x. The set of decisions it needs to take are the values in the array $C(x, s)$ as each element is indexed by the state of the system and the value is the price to be charged or a decision to reject the client. Assume some starting values for each entry of $C(x, s)$.
2) Assume some values for each element of $C(y, s)$ for service provider y .
3) Find the income per unit time at steady state of service provider y, given the current values in $C(x, s)$ and $C(y, s)$. The method is given in section III-C.
4) Repeat step 3 for all possible sets of values of $C(y, s)$.
5) Choose that version of $C(y, s)$ which gives the maximum expected income. Assume that service provider y takes this decision.
6) Now interchange the roles of x and y and let the decision of service provider y found in step 5 become

the decision of service provider x in step 1.

7) Repeat step 2 to step 6 till a Nash Equilibrium condition is satisfied or till all possible values of $C(y, s)$ have been considered. Nash Equilibrium will be reached when, given the decision of service provider 1 is a, the decision with maximum income per unit time of service provider 2 is b and given the decision of service provider 2 is b, the decision with maximum income per unit time of service provider 1 is a.
8) No Nash Equilibrium has been found. Set x to Service provider 1 and y to provider 2. Set $C(x, s)$ to a new set of values and go to step 2 if all combinations of $C(x, s)$ have not been tried out in steps 2 to 6.

C. Finding income per unit time at steady state

The income per unit time at steady state is found by multiplying the steady state probability of being in a particular state by the income at that state and then this is added for every possible state. The method to find the steady state probability is shown in section III-D. The formula to find $I(1)$ is given as equation (1) and its explanation is as follows. An arriving client connects to service provider 1 if it does not reject the client, and it charges less than 2 or 2 rejects the client (this is the first term in the first set). If provider 1 does not reject the client and it charges the same price as provider 2, then with half the probability the arriving client goes to service provider 1 (term 2 in first set). A penalty of $Delay(1, s1) \times P(0)$ has to be incurred and it depends on the number of clients in session (n) is more than B_1 (second set of terms). Its formula is given in Table (II). For the accurate solution, it is written as $Delay(i, m, n)$. Finally, a penalty of $P(1) \times \lambda$ has to be incurred with probability 1 if both providers reject the client and with probability half if provider 1 rejects the client and provider 2 charges the highest possible price.

$$
\begin{aligned}
&I(1) = \\
&\sum_{s1,s2} Pr(s1, s2) \times \{ \\
&\begin{cases} \lambda \times D \times & \\ price(-C(1, s1)) & C(1, s1) \leq 0 \\ & , \{C(2, s2) < C(1, s1) \\ & or\, C(2, s2) = 1\} \\ \frac{\lambda}{2} \times D \times & \\ price(-C(1, s1)) & , C(1, s1) \leq 0 \\ & , C(2, s2) = C(1, s1) \\ 0 & , otherwise \end{cases} \\
&-Delay(1, s1) \times P(0) \\
&-\begin{cases} P(1) \times \lambda & , C(1, s1) = 1, C(2, s2) = 1 \\ P(1) \times \frac{\lambda}{2} & , C(1, s1) = 1 \\ & , C(2, s2) = -(T-1) \\ 0 & , otherwise \end{cases} \quad \}
\end{aligned} \tag{1}
$$

In a similar way, the value of $I(2)$ can be found.

The method of finding D is as follows. As shown in figure 1, when a client connects, he requests for bandwidth for some duration. The expected time for which a client remains in session is $\frac{1}{S(1)}$. The probability that a client disconnects immediately after releasing bandwidth is d. The probability that a client again requests for bandwidth after remaining idle for some time is $(1 - d)$. Therefore the total time that a client spends on consuming bandwidth is

Table II. FUNCTION DEFINITIONS

Function	Definition
$Delay(i, m, n)$	$\begin{cases} 0 & , n \leq B_1 \\ (n - B_1) & , otherwise \end{cases}$
$E(i, m)$	$\sum_n \left(Pri(m, n) \times \begin{cases} B_i & , n > B_i \\ n & , otherwise \end{cases} \right)$
$Delay(i, m)$	$\sum_n \left(Pri(m, n) \begin{cases} n - B_i & , n > B_i \\ 0 & , otherwise \end{cases} \right)$

$$D = \frac{1}{S(1)} \times \left(1 + (1 - d) + (1 - d)^2 + (1 - d)^3 ...\right)$$

On simplification, the value of D is :

$$D = \frac{1}{d \times S(1)} \tag{2}$$

D. Finding steady state probability

The method to find steady state probability is described as follows. Let $(s1, s2)$ represent that service provider 1 is in state $s1$ and service provider 2 is in state $s2$. For any state, the rate of arrival is equal to the rate of departure at steady state. The combined steady state probability $Pr(s1, s2)$ represents the probability that service provider 1 is in state $s1$ and service provider 2 is in state $s2$. The rate of departure is $Pr(s1, s2)\times$ (rate of change of state from $(s1, s2)$ to any other state). The rate of arrival is $\sum_{s3,s4} Pr(s3, s4) \times$(rate of change of state from $(s3, s4)$ to $(s1, s2)$). By solving the equations, steady state probability is found.

IV. APPROXIMATE SOLUTION

The process of finding the Nash Equilibrium using the above analysis requires high memory space and computation time. Therefore, we present an approximate solution which has low space and time complexity. This approximate solution is compared with the accurate solution by running the two solutions for small values of m_{max} and comparing the expected incomes.

In the approximate solution, the state of a service provider is represented by a single integer m, the number of clients connected to the service provider. We do not keep track of the number of clients that are in session or waiting for a session (n in the accurate solution). Instead, we estimate the number of clients that are in session. The method to find the approximate Nash equilibrium is similar to the method used in the accurate solution. There are three differences: the first difference is that the number of solution matrices to be consider are reduced because the state is represented by a single integer, the second difference is that the equations of $Pr()$ are different and the third difference is that the $Delay()$ function's value is different.

A. Steady State Probability and $Delay()$

As already mentioned, in the approximate solution the state of a service provider is represented by a single integer. Therefore, $Pr(m_1, m_2)$ is used to represent the probability of states of the service providers. The method to find the steady state probability for the approximate solution is almost the same as in the accurate solution except that there are a few differences. Among the state changes, only arrival and departure of clients is considered. The opening and closing of a session do not change the number of connected clients and so are ignored. The arrival rate of clients is already known but the departure rate is estimated. The estimated departure rate for service provider i when m_i clients are connected is estimated as $(E(i, m_i) * S(1) * d)$.

Since the state of a service provider is represented by a single integer $Delay(i, s)$ can be written as $Delay(i, m)$. The value of $Delay(i, m)$ and $E(i, m)$ is given in Table (II). The value of $Pri(m, n)$ is obtained using the Engset distribution formula [7].

B. Comparison between accurate and approximate solutions

We compare with two service providers. There are a number of comparisons and in each comparison bandwidths are different. Bandwidth of service providers range from 3 units to 5 units while the maximum number of users, m_{max}, is 8. The mean arrival rate of clients is 0.2 per unit time. The clients mean idle time is 20 units, the mean session duration is 12 units of time, and bandwidth is consumed by every client at 1 unit per unit time. The value of d is 0.4. There are two prices and these are 0.1 and 0.12 per unit data transfer. Penalty for session delay is 0.2 per unit time and penalty for rejection is 1 per rejection. We find the values of the C arrays of the two providers at Nash equilibrium using the method given in section III-B. We then calculate the expected incomes $I(1)$ and $I(2)$ using equation 1.

The result of the comparisons is given in Table III. Each entry of the table contains two values and these values are separated by a comma. The first value is for service provider 1 and the second value is for service provider 2. Column 1 gives the expected income of the service providers when the accurate solution is used and column two gives the expected income of the service providers when the approximate solution is used. The bandwidths of the service providers is given in column 3. The approximate solutions income is at most 15 % less (when the bandwidths are 3 for both; with maximum 8 users, the chances of penalties are high in this case) to almost no difference when the available bandwidth is high (in the case of bandwidths are 5 for both; the chances of penalties are low). So, these preliminary results show that, if highly congested situations can be avoided (which the ISPs will strive to do as otherwise penalties will increase), the approximate solution will perform as well as the accurate solution.

Table III. COMPARISON BETWEEN ACCURATE AND APPROXIMATE SOLUTIONS

Accurate solution income	Approximate solution income	Bandwidth
0.141, 0.141	0.120, 0.120	3,3
0.152, 0.228	0.132, 0.227	3,4
0.154, 0.306	0.138, 0.303	3,5
0.245, 0.245	0.237, 0.237	4,4
0.249, 0.292	0.230, 0.293	4,5
0.283, 0.283	0.283, 0.283	5,5

V. CONCLUSION

Our model incorporates some of the ideas proposed by other researchers. We use dynamic pricing and we impose penalties as a way of providing guarantees. We use the game theoretic concept of Nash Equilibrium to find the best pricing strategy. No paper has considered the situation depicted in our scheme, which we feel models real situations fairly accurately. Our scheme and the solutions are therefore novel.

We have presented a Nash Equilibrium solution between two internet service providers which maximizes the income of each of the service providers. But the solution has high complexity and so we have also proposed an approximate solution of lower complexity. Our results show that the approximate method gives results very close to the accurate method.

The approximate method can be applied to multiple service providers also. As future work we intend to explore the range of solutions that we get when the number of providers are more than two, and when the bandwidths of providers are different. We also intend to examine if we can show the existence of a Nash Equilibrium in all cases, and whether such an equilibrium is unique.

REFERENCES

[1] I. J. B. F. Adan and J. A. C. Resing. *Queueing theory. Lecture notes*. Eindhoven University of Technology, ftp://ftp.win.tue.nl/pub/stochor/queueing.pdf., 2001.

[2] C. A. Gizelis and D. D. Vergados. A survey of pricing schemes in wireless networks. *IEEE Communications Surveys and Tutorials*, 13(1):126–145, 2011.

[3] J. Kosinski, D. Radziszowski, K. Zielinski, S. Zielinski, G. Przybylski, and P. Niedziela. Definition and evaluation of penalty functions in sla management framework. In *Networking and Services, 2008. ICNS 2008. Fourth International Conference on*, pages 176–181, 2008.

[4] H. Pervaiz and J. Bigham. Game theoretical formulation of network selection in competing wireless networks: An analytic hierarchy process model. In *Next Generation Mobile Applications, Services and Technologies, 2009. NGMAST '09. Third International Conference on*, pages 292–297, 2009.

[5] R. S. Randhawa R. Garg. A sla framework for qos provisioning and dynamic capacity allocation. In *10th International Workshop on Quality of Service (IWQoS 2002)*, pages 129–137, May 2002.

[6] Ramona Trestian, Olga Ormond, and Gabriel-Miro Muntean. Game theory-based network selection: Solutions and challenges. *Communications Surveys Tutorials, IEEE*, 14(4):1212–1231, 2012.

[7] Moshe Zukerman. *Introduction to queueing theory and stochastic teletraffic models*. 2000.